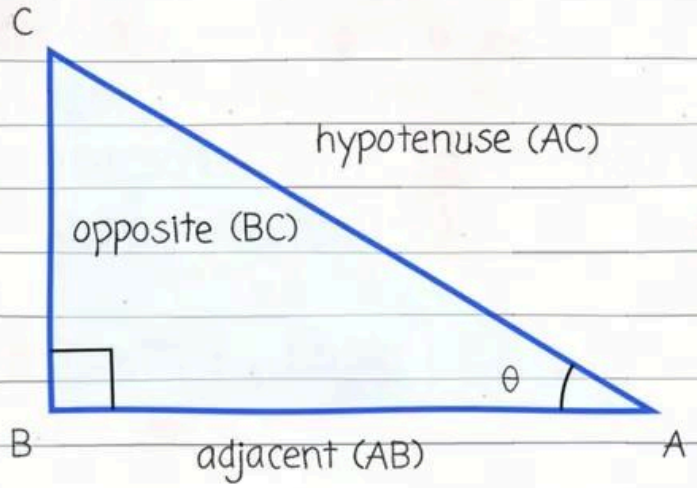


Right angle at B. Sides named for the angle θ at A

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Ratios depend only on the angle, not on the size of the triangle

* Trick : SOH CAH TOA: Sin = Opp/Hyp, Cos = Adj/Hyp, Tan = Opp/Adj

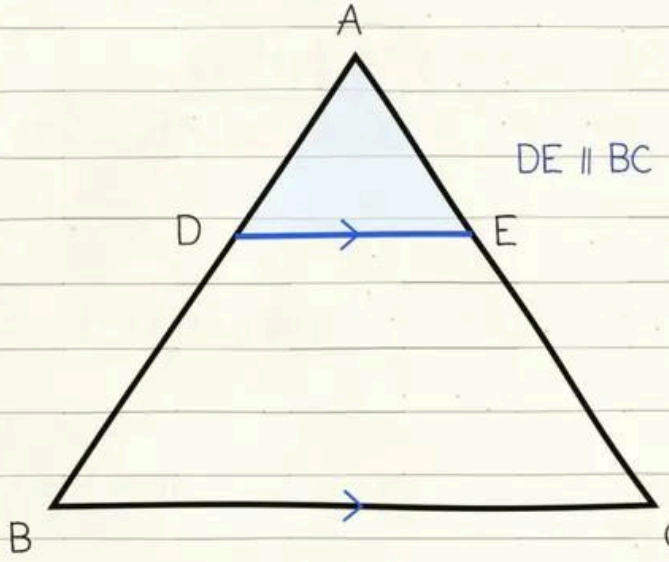
- Same shape, not necessarily the same size. Congruent triangles are always similar.
- Write the vertices in matching order: $\triangle ABC \sim \triangle PQR$ means $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$.

- So $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$.
- The shared ratio is called the scale factor.

⚜ Careful : Pairing the wrong sides. Always match sides opposite to equal angles.

⚜ 2. Basic Proportionality Theorem (BPT)

BPT (Thales theorem) : A line drawn parallel to one side of a triangle, cutting the other two sides at distinct points, divides those sides in the same ratio.



DE ∥ BC, so $\frac{AD}{DB} = \frac{AE}{EC}$ and $\triangle ADE \sim \triangle ABC$

Chapter - 4

Quadratic Equations

Most asked in boards

- Speed-distance-time word problems (plane delayed, train, boat in stream, return journey)
  - Asked : 2016, 2017, 2018, 2019, 2020, 2024, 2026 (3 to 5 marks)
- Solve a quadratic by factorisation (splitting the middle term) or by the quadratic formula
  - Asked : 2016, 2017, 2019, 2020, 2022, 2023, 2024 (1 to 4 marks)
- Nature of roots using the discriminant; find k for equal / real / no real roots
  - Asked : 2016, 2017, 2019, 2023, 2024, 2025 (1 to 5 marks)
- Area/geometry word problems (rectangle or right-triangle sides, two squares from area and perimeter, tiles)
  - Asked : 2016, 2017, 2019, 2024, 2025, 2026 (4 to 5 marks)
- Number, digit, reciprocal and marks word problems
  - Asked : 2017, 2019, 2022, 2023 (3 to 4 marks)

1. Standard form and roots

Quadratic equation : An equation that can be written as $ax^2 + bx + c = 0$, with a, b, c real and $a \neq 0$.

- A root (solution) is a value of x that makes the equation true.
- A quadratic has at most 2 roots.
- First bring everything to one side so the right side is 0.

- On the graph it is the point where the two lines meet.
- Consistent: has at least one solution. Inconsistent: no solution.



Intersecting
1 solution

Parallel
no solution

Coincident
infinitely many

Three graph cases for a pair of linear equations

Graph	Condition	Solutions	Type
Lines intersect	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	exactly one	consistent
Lines coincide	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	infinitely many	consistent (dependent)
Lines parallel	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	none	inconsistent

✯ Trick : Compare the ratios in the order a, b, c. First two equal and third different = parallel = no solution.

Q2. The sum of the first 6 terms of an AP is 57 and the sum of its first 10 terms is 155. Find the AP and its 15th term.

☆ Asked : 2016, 2017, 2019, 2020, 2022, 2023, 2024 (3 marks)

Ans. $S_6 = \frac{6}{2}(2a + 5d) = 57$, so $2a + 5d = 19$. (1)

$S_{10} = \frac{10}{2}(2a + 9d) = 155$, so $2a + 9d = 31$. Subtracting: $4d = 12$, so $d = 3$ and $a = 2$. (1)

The AP is 2, 5, 8, ... and $a_{15} = 2 + 14 \times 3 = 44$. (1)

Q3. Find the sum of all the multiples of 7 that lie between 100 and 300.

☆ Asked : 2018, 2020, 2022, 2024, 2025, 2026 (3 marks)

Ans. The multiples are 105, 112, ..., 294 ($7 \times 15 = 105$ and $7 \times 42 = 294$). (1)

$294 = 105 + (n - 1) \times 7$, so $n - 1 = 27$ and $n = 28$. (1)

Sum $= \frac{28}{2} \times (105 + 294) = 14 \times 399 = 5586$. (1)

Q4. The sum of three numbers in AP is 27 and their product is 693. Find the numbers.

☆ Asked : 2016, 2018, 2020, 2024 (5 marks)

Ans. Let the numbers be $a - d$, a and $a + d$. (1)

Sum: $3a = 27$, so $a = 9$. (1)

Product: $(9 - d) \times 9 \times (9 + d) = 693$, so $81 - d^2 = 77$. (1)

$d^2 = 4$, so $d = 2$ or $d = -2$. (1)

The numbers are 7, 9, 11 (or 11, 9, 7 when $d = -2$). (1)

* Careful : Do not say 6^n cannot end in 0 'because it is even'. Give the reason: no factor 5 in $2^n \times 3^n$.

2. HCF and LCM by prime factorisation

HCF = product of the smallest power of each common prime

LCM = product of the greatest power of every prime that appears

Works for two or three numbers

- Step 1: write each number as a product of primes.
- Step 2: list the primes; pick smallest power (HCF) or greatest power (LCM).
- Step 3: multiply them out.
- LCM given as prime powers? Just take the highest power of each prime.

HCF	LCM
Greatest number that divides all	Smallest number divisible by all
Only common primes	All primes of all numbers
Smallest powers	Greatest powers
Used for: largest equal groups, cutting into equal pieces	Used for: meeting again together, bells, traffic lights

- Distance of P(x, y) from the x-axis = |y|, from the y-axis = |x|.
- Reflection of (x, y): in the x-axis (x, -y), in the y-axis (-x, y), in the origin (-x, -y).
  → P(-6, 8): distance from the x-axis is 8, from the y-axis is 6, from the origin 10.

☆ Asked : 2020, 2023, 2025, 2026

☆ Careful : Distance from the x-axis is |y|, the y-coordinate without its sign (for (3, -5) it is 5), not the x-coordinate. Distances are never negative.

☆ 2. Distance formula

$$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

Distance from the origin: $OP = \sqrt{x^2 + y^2}$

From the Pythagoras theorem on the right triangle formed by the two points

- Subtract in the same order for both coordinates. The square removes the sign.
- Keep the root till the end. Simplify with perfect squares only.
  → A(3, -2), B(-1, 1): $AB = \sqrt{16 + 9} = 5$.

☆ Asked : 2018, 2019, 2020, 2022, 2023, 2024, 2026

Unknown coordinate

- 'Equidistant from A and B': set PA = PB, square both sides (drop the roots).
- 'On the x-axis': take P = (x, 0). 'On the y-axis': take P = (0, y).

- Sum, difference or product of two irrationals can be rational or irrational.
  → $\sqrt{2} \times \sqrt{2} = 2$ (rational). $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$ (rational). $\sqrt{2} + \sqrt{3}$ stays irrational.

☆ Asked : 2019, 2023, 2024, 2025

Key result used in every proof : If a prime p divides a^2, then p divides a.

Proof that $\sqrt{p}$ is irrational (p prime): steps

1. Assume the opposite: $\sqrt{p} = \frac{a}{b}$ with a, b co-prime integers, $b \neq 0$.
2. Square: $a^2 = p\,b^2$. So p divides a^2, hence p divides a.
3. Write $a = pc$. Then $p^2c^2 = p\,b^2$, so $b^2 = p\,c^2$.
4. So p divides b^2 and hence b.
5. Now a and b both have the factor p. This contradicts 'co-prime'.
6. So the assumption was wrong. $\sqrt{p}$ is irrational.

Eg. Prove that $3 + 2\sqrt{5}$ is irrational. (Given: $\sqrt{5}$ is irrational.)

Sol : Assume $3 + 2\sqrt{5} = \frac{a}{b}$, a and b integers, $b \neq 0$

$$2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}, \text{ so } \sqrt{5} = \frac{a - 3b}{2b}$$

RHS is a ratio of integers, so rational
But $\sqrt{5}$ is irrational. Contradiction.
Hence $3 + 2\sqrt{5}$ is irrational.

☆ Asked : 2016, 2017, 2018, 2019, 2020, 2023, 2024, 2025, 2026